International Color Consortium®

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Weighting functions for calculation of XYZ tristimulus values

Introduction

XYZ tristimulus values are calculated by summing the product of the measured stimulus and a colour matching function $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ at each wavelength interval. For reflectance measurements, the stimulus is the product of reflectance factor $R(\lambda)$ and the selected illuminant $S(\lambda)$ which represents the intended viewing condition [1]. Thus XYZ tristimulus values are found by equation 1:

$$X = k\sum_{l}^{u}\bar{x}(\lambda)S(\lambda)R(\lambda)$$
$$Y = k\sum_{l}^{u}\bar{y}(\lambda)S(\lambda)R(\lambda) \qquad (1)$$
$$Z = k\sum_{l}^{u}\bar{z}(\lambda)S(\lambda)R(\lambda)$$

where *u, l* are the upper and lower bounds of the wavelength range in $S(\lambda)$, $R(\lambda)$ and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, and *k* is a normalising constant.

CIE recommends computing XYZ at 1nm or 5nm intervals, and defines colour matching functions and standard illuminants at these intervals, over the range 360-780nm.

Many of the measurement instruments in common use in colour management output spectral data at 10nm intervals. While in principle it is possible to simply interpolate measurements from 10 to 5nm, this is not good practice since a) different methods of interpolation will give different results; and b) any bandpass correction applied to the 10nm data may not be correct for a different interval. Instead it is recommended to use weighting functions which combine both observer and illuminant. This also simplifies the calculation in eqn 1 a little, since it now becomes (in matrix form):

$$\boldsymbol{X} = \mathbf{R}\boldsymbol{w}_x$$
$$\boldsymbol{Y} = \mathbf{R}\boldsymbol{w}_y \qquad (2)$$
$$\boldsymbol{Z} = \mathbf{R}\boldsymbol{w}_z$$

where $\mathbf{R}$ is the array of reflectance values, with one row per sample, and the weight vectors $\boldsymbol{w}_{x,y,z}$ are column-wise [2].

Wavelength range

Common measurement instruments provide data in the range 380-730nm, or occasionally 400-700nm. These ranges, while narrower than those recommended by CIE, do not greatly impact the resulting XYZ values, since the sensitivity of the human visual system (and hence the values of the colour matching functions) is very low outside the range 400-700nm. When using the weighting tables as described above, adding the missing measurement data by linear interpolation from the values at the ends of the measured range gives the best performance. Another commonly-used method is to simply extend the values at the ends of the measurement ranges.

Methods of calculating weighting functions

Several different methods for calculating weighting functions exist. ASTM used a well-established method to compute weighting functions for both CIE 1931 and 1964 observers and a wide variety of illuminants, and these were published as tables in ASTM E.308 [3].

ISO 13655 [4] defines computation procedures for use in graphic technology, and ICC recommendations on measurement are based on this standard. It specifies the D50 illuminant and the CIE 1931 observer, and provides the corresponding 10 and 20nm weighting functions from ASTM E.308.

Li, Luo, Melgosa and Pointer (2016) evaluated several different methods of calculating weighting functions [5]. They used 3517 reflectance measurements of Pantone and Munsell chips, and calculated the difference between 1nm calculations and 10nm weighting tables. They found that the Li-Wang-Luo method clearly gave the best results, and was also relatively straightforward to calculate. The algorithm is given in detail in reference [5]. It takes as input the 1nm values of $S(\lambda)$, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, together with the desired interval, and outputs the weights $W_X(\lambda)$, $W_Y(\lambda)$, $W_Z(\lambda)$ which can be used in eqn 2.

Average CIELAB colour differences for the three illuminants (D65, D50 and A) and two standard observers (1931 and 1964) were 0.001 for the LWL method and 0.003 for the ASTM weighting function tables. For most practical applications these differences are not important, and there is no reason to change from the procedures recommended in ISO 13655. The finite precision of computation also limits the possible degree of accuracy, and ICC.1 users will see little or no difference between the LWL and ASTM methods.

Use of weighting functions in ICC.2

In ICC.2 a higher level of precision is possible, through the use of floating point data types and functional calculations. In addition, ICC.2 is not limited to D50 and the 1931 observer but supports connection of arbitrary colorimetry through Profile Connection Conditions. This can include both 1931 and 1964 observers, the more recent 2006 observer [6][7] and even custom observers (for example, where an individual's colour matching function has been measured or assigned to a category of observers) [8]; and it can include any arbitrary illuminant, such as the CIE indoor daylight [9] and LED illuminants [1], or the D50noUV illuminant proposed in the revision of ISO 3664 currently in development in ISO TC130.

ICC provides weighting function tables (computed by NTNU post-doc Tanzima Habib) for 1931 and 1964 observers, and for illuminants D50, D65, A, LED-B1 and F11 [10]. These can be used in different ways, including:

- To compute tristimulus values offline before using them in an ICC.2 workflow
- To incorporate in an ICC.2 profile which converts between spectral and colorimetric data
- To reference by URL in a workflow where the illuminant and observer are selected as a CMM control option

It is also intended to add weighting tables for CIE 2006 LMS and tristimulus functions when available.

Feedback on the use of these new weighting tables is welcomed. Currently CIE TC1-102 is evaluating methods of tristimulus integration for a future CIE recommendation, and any feedback based on practical experience with the tables will be passed on to the TC.

See also ICC Technical Note TN-01-2025 on Recommendations on calculation of XYZ tristimulus values [11].

References

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